EX (cont):

What should be the stop band cutoff frequency of the high pass filter

$$\alpha = -0.382 = -\frac{\cos\left(\frac{\omega_s' + \omega_s}{2}\right)}{\cos\left(\frac{\omega_s' - \omega_s}{2}\right)}$$

$$= -\frac{\cos\left(\frac{0.3\pi + \omega_s}{2}\right)}{\cos\left(\frac{0.3\pi - \omega_s}{2}\right)}$$

$$\boxed{\omega_s = 0.4586\pi}$$

This is a transendental equ which can be solved by some iteration technique.

★ verify the coefficient α is satisfying the transformation form LP to HP. ★

$$-\omega_s' = -\angle G(e^{-j\omega})$$

$$\therefore \quad \omega_s' = +\angle G(e^{-j\omega})$$

$$= \angle \frac{\alpha + e^{-j\omega_s}}{1 + \alpha e^{-j\omega_s}}$$

$$= \angle \frac{-0.382 + e^{-j0.4586\pi}}{1 - 0.382 e^{-j0.458\pi}}$$

$$= 0.3\pi$$

4.3 FIR Filters

FIR filter Structures.

An FIR filter has a system function of the following filter

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots\dots b_{N-1} z^{-N-1}$$

$$= \sum_{n=0}^{N-1} b_n z^{-n}$$

The coorisponding impulse responce

$$h[n] = \begin{cases} b_n , & 0 \le n \le N-1 \\ 0 , & \text{Otherwise} \end{cases}$$

$$= b_0 + b_1 \delta[n-1] + \dots + b_{N-1} \delta[n-N+1]$$

The coorisponding D.E.

$$y[n] = b_0 x[n] + b_1 x[n-1] + \dots$$
$$+ b_{N-1} x[n-N+1]$$

The filter order is N-1, while the length of the filter is N, the FIR filter structure is always stable. As the poles are at zero and it is relatively simple as compared to IIR

Moreover, the FIR filter can be designed to have a linear phase response which is desirable in some applications (Dubbing)

The possible four structures of FIR filters are

(a) Direct form: in the case of DF II it is implemented directly.

(b) Cascade form: The system function (1) is factored into second order, which are then ~~implemented~~ by cascade connection

(c) Linear Phase form: When an FIR filter has a linear phase response its impulse has symmetry which can reduce calculations. by 1/2.

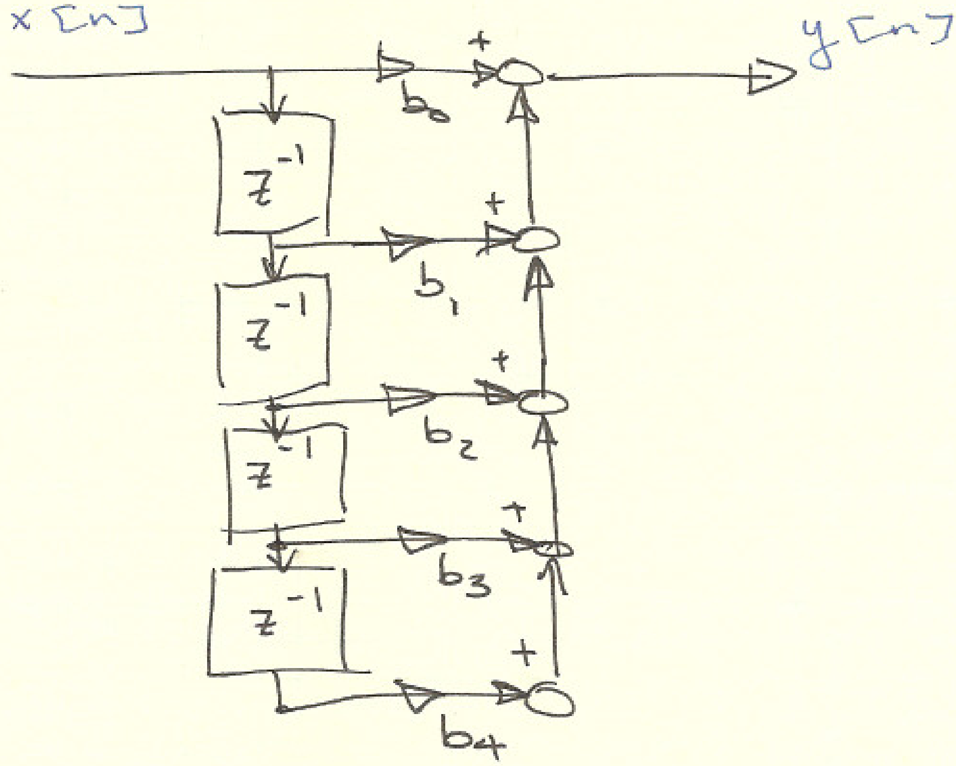
(d) Freq Sampling form: This structure is based on the DFT values of impulse response h[n] It leads to a parallel structure.

EX: Consider a 4th order FIR filter as follows.

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$+ b_3 x[n-3] + b_4 x[n-4]$$

(A)



(B) Cascade form

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{N-1} z^{-N-1}$$

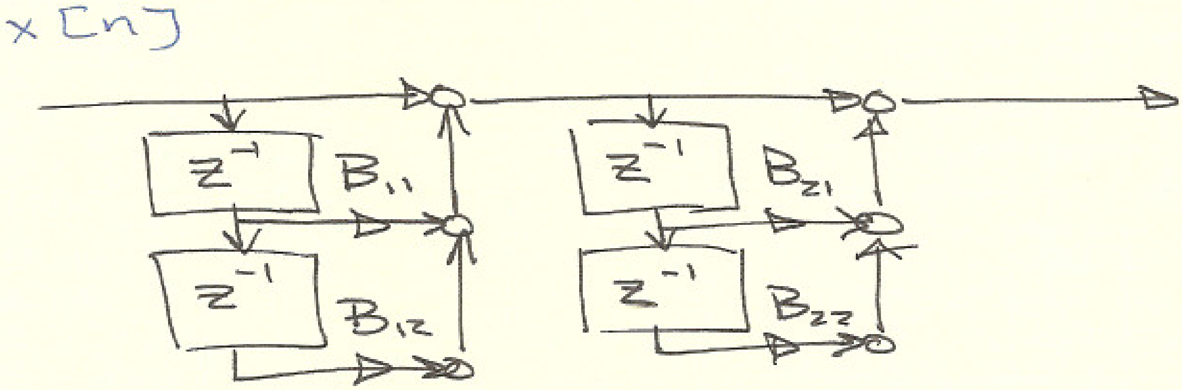
$$= b_0 \left[1 + \frac{b_1}{b_0} z^{+1} + \dots + \frac{b_{N-1}}{b_0} z^{-N-1} \right]$$

$$= b_0 \prod_{k=1}^{\lfloor N/2 \rfloor} \left(1 + B_{k,1} z^{-1} + B_{k,2} z^{-2} \right)$$

k^{th} stage.

delay #

for cascade form $N=5$



* only need to know the process, not how to calculate $B_{x,x}$. *

Ⓒ Linear Phase form.

This means that the filters phase response is a linear function of frequency.

$$\therefore \quad \angle H(e^{j\omega}) = \beta - \alpha\omega$$

$$\boxed{-\pi < \omega < \pi}$$

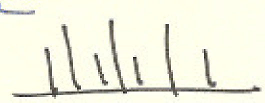
Where $\beta = 0$ or $\pm \pi/2$

$\alpha =$ constant.

For causal FIR filter the linear phase cond imposes the following symmetry condⁿ:

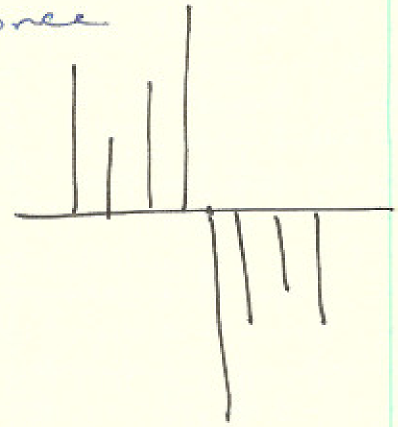
* $h[n]\ h[N-1+n]$, $\beta = 0$

Symmetric impulse Response $\quad 0 \le n \le N-1$



* Antisymmetric impulse responce

$$h[n] = -h[N-1-n]$$



$$\beta = \pm \pi/2$$

$$0 \leq n \leq N-1$$

$$= 2e^{-jw\left(\frac{N-1}{2}\right)} \sum_{i=0}^{\frac{N}{2}-1} h[i] \cos\left(w\left(\frac{N-1}{2}-i\right)\right)$$

$w = 0 \longrightarrow H_r(w)$ may be $\emptyset$

This filter is not good for HP or BS filter.

* Type-3 ($h[n]$ is anti-symmetric & N-odd)

$w = 0, \pi \longrightarrow H_r(w) = 0$

* Type-4 ($w = 0$